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CCPR K3 Key Comparison of Luminous Intensity

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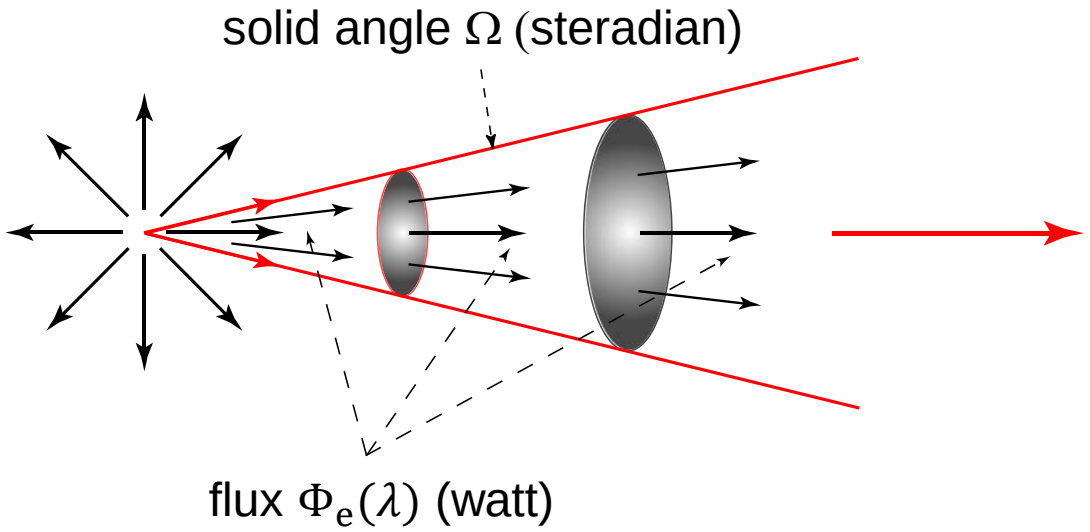
CORM 2019 Annual Technical Conference
2019-October-28

Intensity

Radiant Intensity

$$I_e(\lambda) = \frac{\Phi_e(\lambda)}{\Omega} = \frac{\text{radiant flux}}{\text{solid angle}}$$

unit = watt per steradian



Intensity

Luminous Intensity

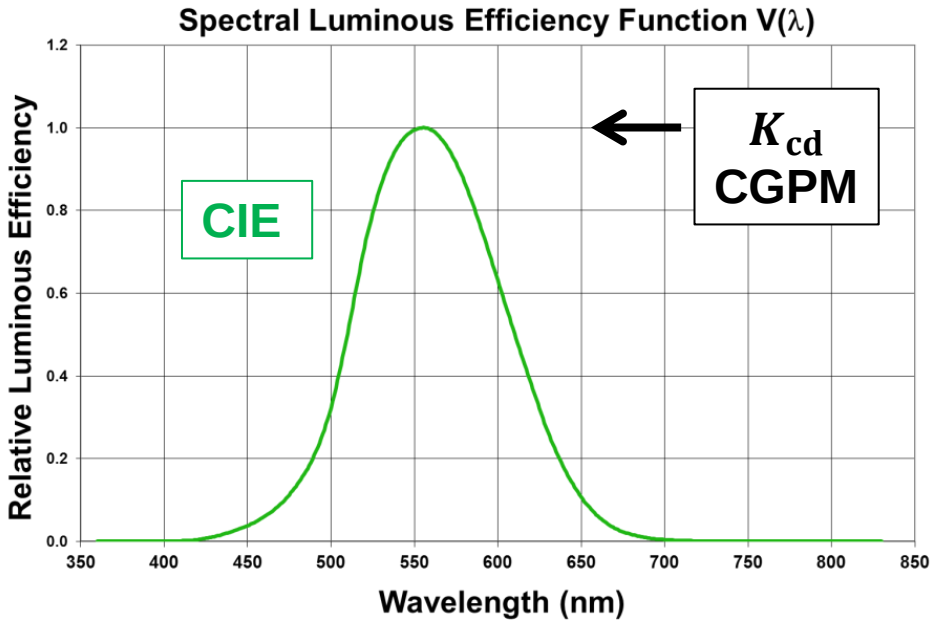
$$I_v = \frac{\Phi_v}{\Omega} = \frac{\text{luminous flux}}{\text{solid angle}}$$

unit = candela = lumen per steradian

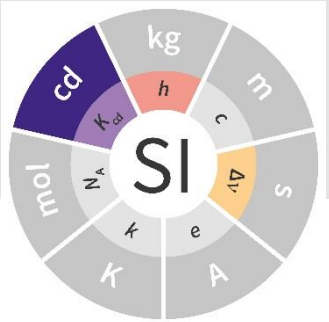
$$\Phi_v = K_{cd} \int_{360\ nm}^{830\ nm} V(\lambda) \cdot \Phi_e(\lambda) \cdot d\lambda$$

unit = lumen

$$K_{cd} = 683\ \frac{\text{lumen}}{\text{watt}}$$

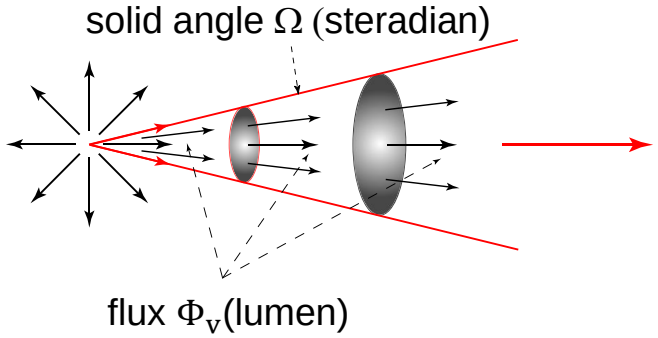


Intensity



Luminous Intensity, SI unit candela

$I_v = \frac{\Phi_v}{\Omega}$ unit = lumen per steradian = candela



CGPM definition:

The candela, symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540 x 10^{12} Hz, K_{cd}, to be 683 when expressed in the unit lm W^{-1}, which is equal to cd sr W^{-1}, or cd sr kg^{-1} m^{-2} s^3, where the kilogram, metre and second are defined in terms of *h*, *c* and $\Delta\nu_{Cs}$.

https://www.bipm.org/en/measurement-units/base-units.html

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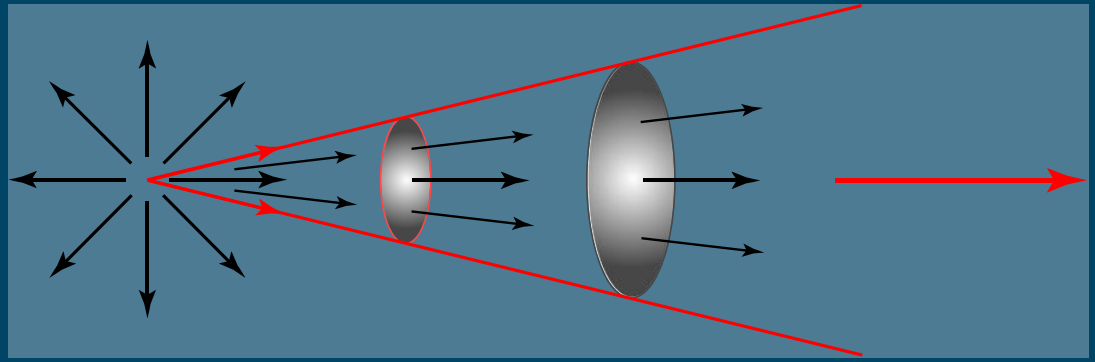

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Comparison Organisation

- Selection of participants, artifacts and protocol

Comparison Procedures

- Comparison measurements and measurement verification
- Data analysis and comparison of participant SI candela realisations
- Write the report

CCPR Key Comparison CCPR-K3.2014

Comparison Organisation

- Selection of NRC as pilot
- Selection of participants (12 max)
- Task Group
  - Selection of artifact
    - Lamp vs photometer: standards-quality incandescent lamps
    - Type of lamp: Incandescent (Osram Wi41/G and NPL/Polaron heavy current)
    - Type of comparison (star type: participant – pilot – participant)
      - Standard lamps are fragile and expensive
  - Draft the technical protocol (artifact transportation, measurement reporting, uncertainties, etc.)
- Register the comparison: CCPR-K3.2014

RMO Group	RMO Group Members	Maximum Number of Participants
Group 1	EURAMET+COOMET	6
Group 2	APMP+AFRIMETS	4
Group 3	SIM	2

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Comparison Organisation

- Selection of participants

NMI	Country
NMISA	South Africa
NIM	China
NMIA	Australia
NMIJ	Japan
IO-CSIC	Spain
LNE-CNAM	France

NMI	Country
METAS	Switzerland
NPL	UK
PTB	Germany
VNIIOFI	Russia
NIST	USA
NRC	Canada

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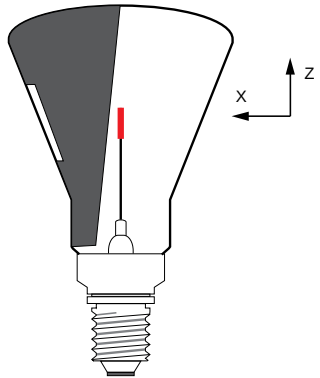
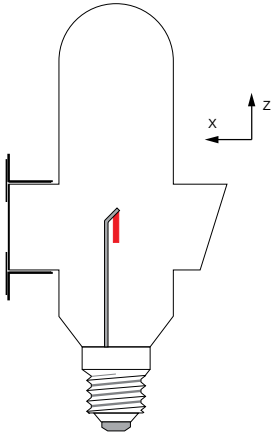
Comparison Artifact

- Type of lamp: Incandescent (Osram Wi41/G and NPL/Polaron heavy current)





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Comparison Procedures ● Measurements

- Comparison measurements
  - Each participant supplied their own calibrated (~6) lamps (ship or hand-carry)
  - NRC received and measured ~70 comparison lamps
  - Each participant re-measured their lamps
- Measurement verification and artifact certification
  - Each participant compares before and after shipment measurements
  - NRC provides relative data for all the artifacts of each participant
  - Removal of unstable artifacts => final comparison artifacts

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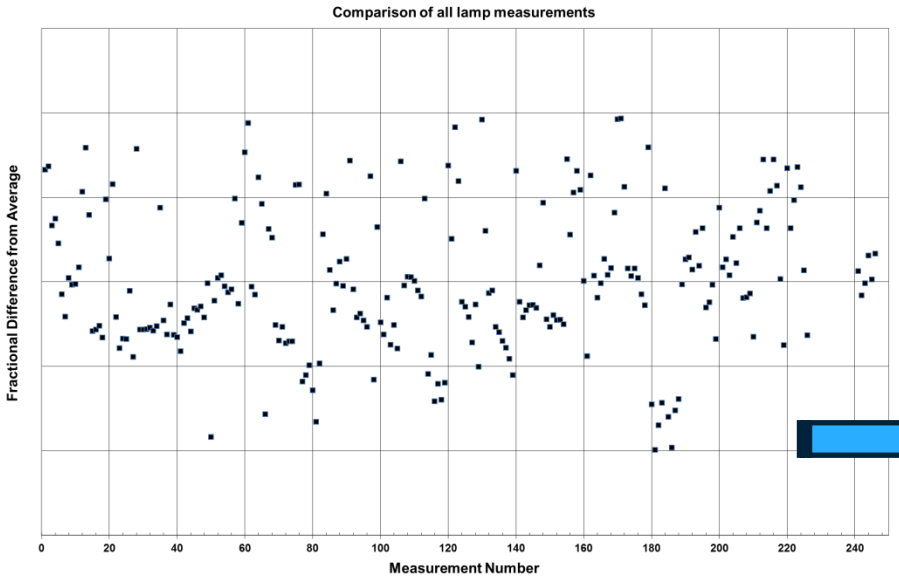
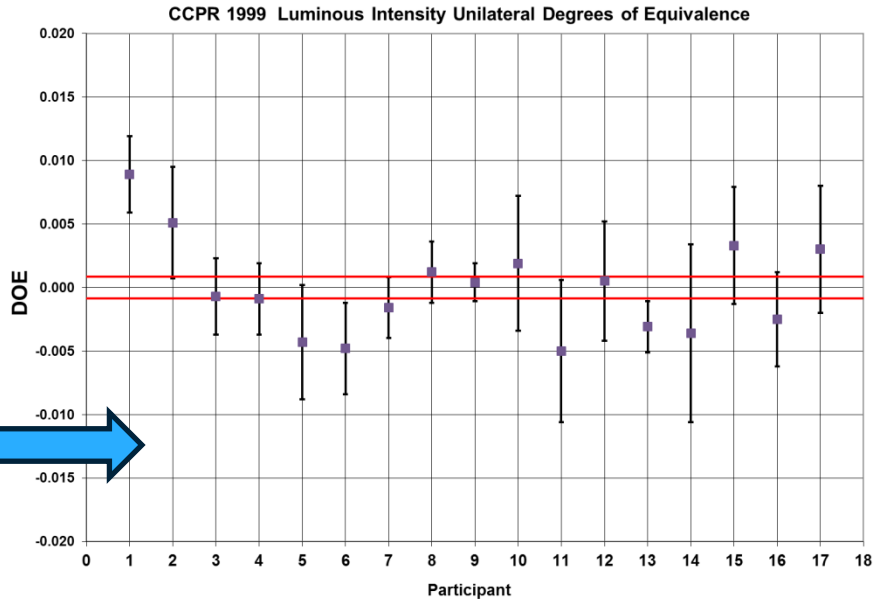
Comparison Procedures ● Data Analysis





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Comparison Procedures ● Analysis

- Data analysis
  - Determine final NRC measurement value for each artifact
  - Determine final NRC measurement value for each participant
- Comparison of participant SI candela realisations
  - KCRV (Key Comparison Reference Value)
    - Weighted mean with cut-off
  - Identification of ‘outliers’: deviation from KCRV greater than 6 times their uncertainty
  - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$
- All this requires an uncertainty analysis (NRC and Participant measurements)

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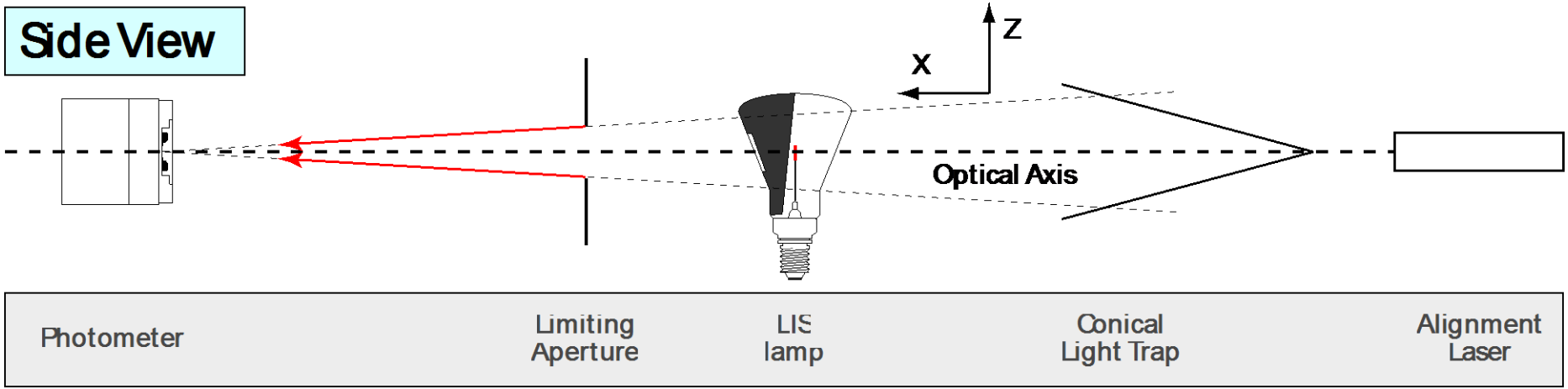
Comparison Procedures ● Measurements at pilot (NRC)

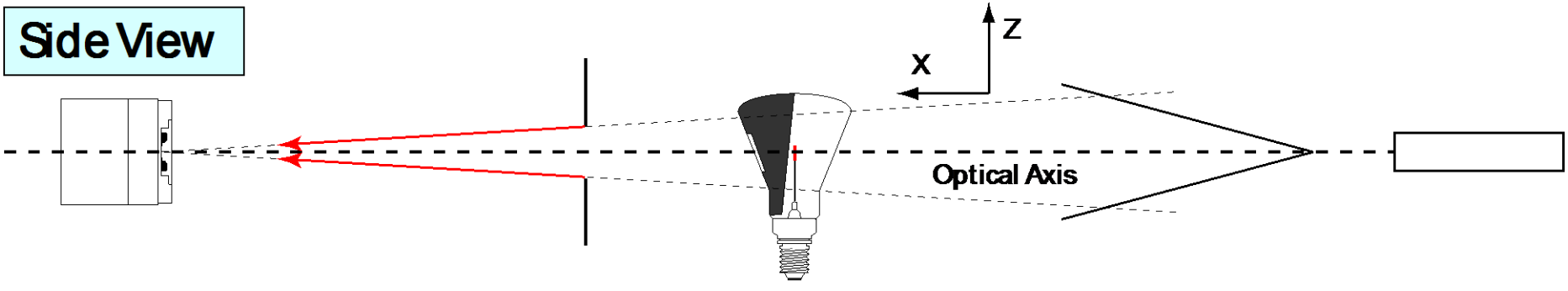
- Comparison of all artifacts under identical measurement configuration



CCPR Key Comparison CCPR-K3.2014

Comparison Procedures ● Measurements at pilot (NRC)

- Comparison of all artifacts under identical measurement configuration



$$R_{i,j,m} = \frac{I_{v(i,j)}}{V_{i,j,m}} \left(\frac{\text{cd}}{\text{volt}} \right)$$

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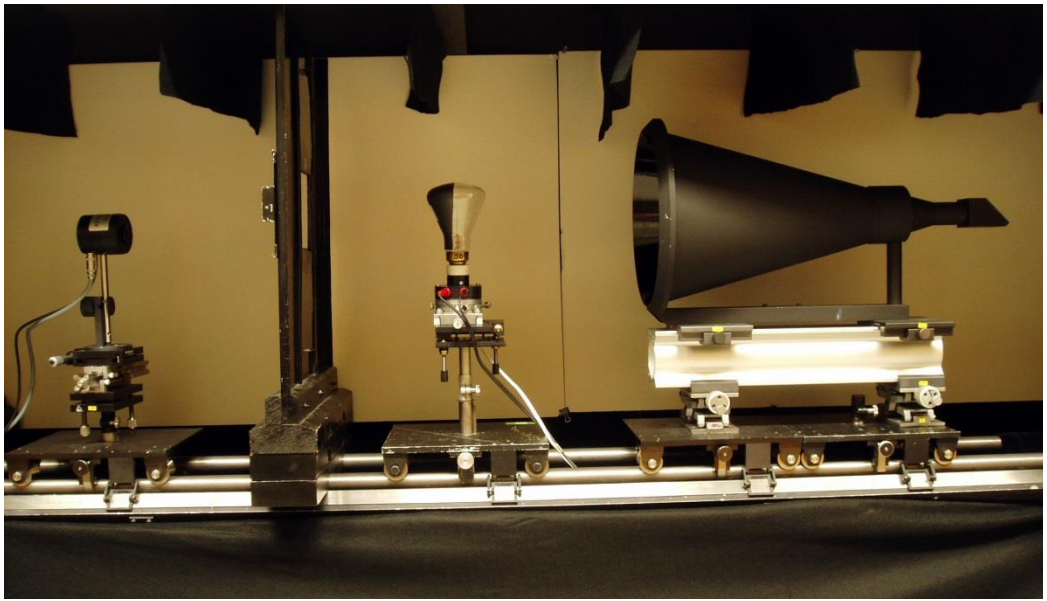
Comparison Procedures ● Measurements at pilot (NRC)

- Comparison of all artifacts under identical measurement configuration
- $d \sim 3.2\ m$
- 3 photometers
- ≥ 2 measurements/lamp
- ~ 250 measurements
- ~ 2 months



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Comparison Procedures ● Measurements at pilot (NRC)

- **How accurate is the comparison?**
- Sources of Uncertainty ● $u(V_{i,j,m})$ ● (~15!)
  - NRC Optical Coordinate System (2)
  - NRC Photometer (5)
  - Participant Lamps
    - Electrical (4)
    - Optical (3)
    - Photometric (1)

  Consider:
  - In 3D space there are 6 variables: 3 spatial and 3 angular
  - Lamp output: % change ≈ 7 times % change in lamp current
    - Am I operating the lamp electricals to the same standards as the participant?
  - How/with what do I ensure stability over 2 months of measurements?

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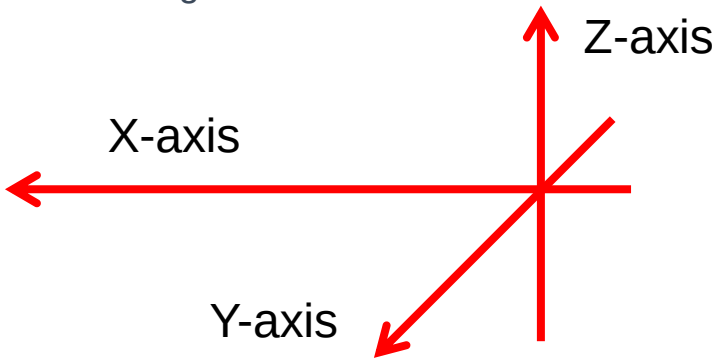
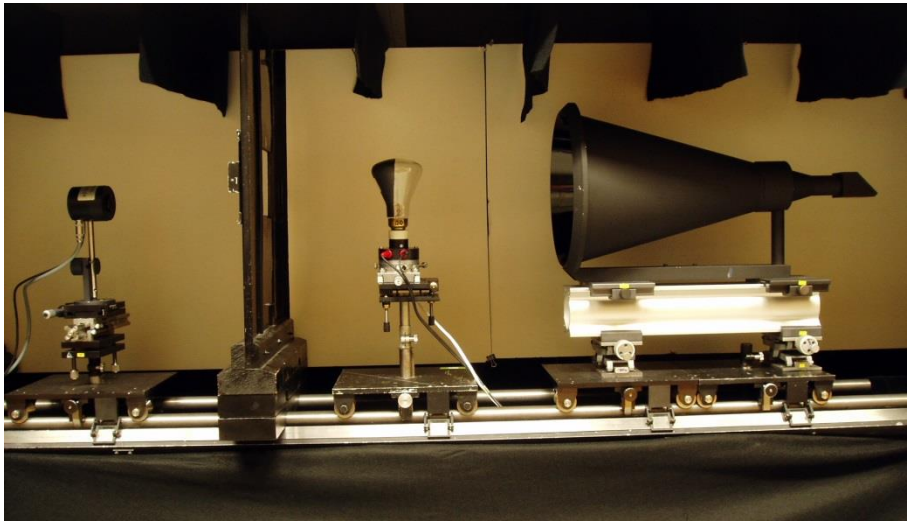
Comparison Procedures ● Measurements at pilot (NRC)

- Sources of Uncertainty ● $u(V_{i,j,m})$
  - NRC Optical Coordinate System (2)
    - Starting line is X-axis (laser beam)
    - Alignment of Y-axis to X-axis (laser)
    - Alignment of Z-axis to XY axes

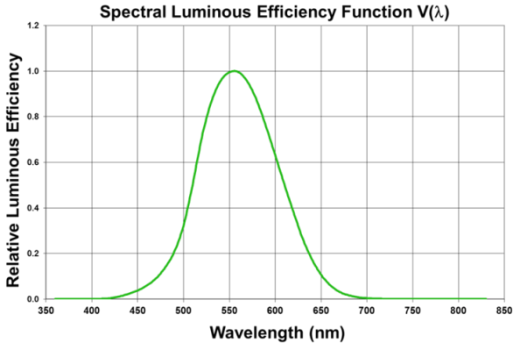




CCPR Key Comparison CCPR-K3.2014

Comparison Procedures ● Measurements at pilot (NRC)

- Sources of Uncertainty ● $u(V_{i,j,m})$
  - NRC Optical Coordinate System (2)
  - NRC Photometer (5)
    - Spectral Mismatch Error
      - $F^* = \frac{\int_{360\,nm}^{830\,nm} P_e^T(\lambda)\cdot V(\lambda)\cdot d\lambda}{\int_{all\ wavelengths} P_e^T(\lambda)\cdot R(\lambda)\cdot d\lambda} \frac{\int_{all\ wavelengths} P_e^S(\lambda)\cdot R(\lambda)\cdot d\lambda}{\int_{360\,nm}^{830\,nm} P_e^S(\lambda)\cdot V(\lambda)\cdot d\lambda}$
    - Responsivity Drift (<u>what</u> is constant over the 2 months of measurements?)
    - Signal Noise (fluctuations)
    - Alignment to optical axis (Y-Z centre)
    - Alignment to optical axis (Y-Z angular)



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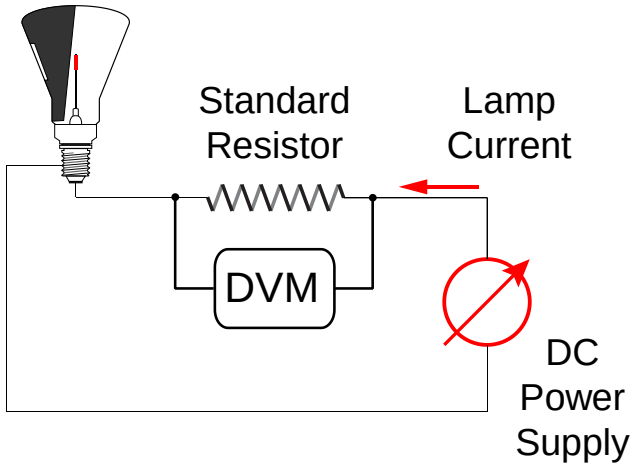
Comparison Procedures • Measurements at pilot (NRC)

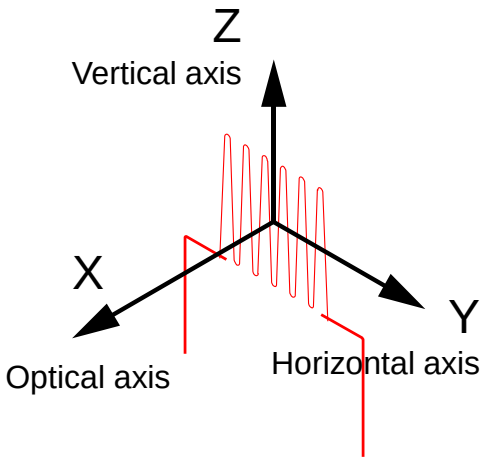
- Sources of Uncertainty • $u(V_{i,j,m})$
  - NRC Optical Coordinate System (2)
  - NRC Photometer (5)
  - Participant Lamps
    - Electrical (4)
      - Standard Resistor calibration (lamp current measurement)
      - DVM voltage calibration (lamp current measurement)
      - Lamp current setting
      - Lamp current fluctuations
      - % change in lamp output is approximately 7 times % change in lamp current
    - Optical (3)
    - Photometric (1)



CCPR Key Comparison CCPR-K3.2014

Comparison Procedures ● Measurements at pilot (NRC)

- Sources of Uncertainty ● $u(V_{i,j,m})$
  - NRC Optical Coordinate System (2)
  - NRC Photometer (5)
  - Participant Lamps
    - Electrical (4)
    - Optical (3)
      - Vertical filament plane (parallel to Z-axis, rotation about Y-axis)
      - Vertical filament plane (parallel to Y-axis, rotation about Z-axis)
      - Lamp to photometer distance (photometer signal $\propto {}^{1}/_{d^2}$)
    - Photometric (1)
      - Lamp output fluctuations



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Comparison Procedures ● Measurements at pilot (NRC)

- Sources of Uncertainty ● Summary ● $u(V_{i,j,m})$
  - 4 predominant sources of uncertainty:

Source of Uncertainty	Type	Relative Standard Uncertainty
NRC Photometer		
Spectral Mismatch Error	B	0.01%
Responsivity Drift	A	0.05%
Participant Lamps (optical)		
Vertical Filament Plane	A	0.01%
Lamp-to-Photometer distance	A	0.03%

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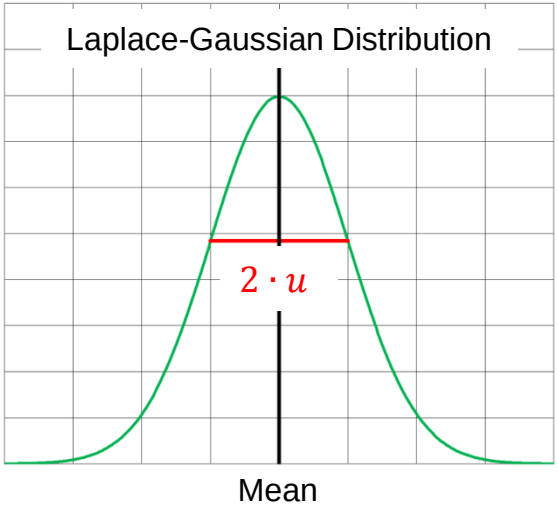
Comparison Procedures ● Data Analysis

- Sources of Uncertainty
  - 3 sources:
    - Participant LI values
    - NRC comparison measurements
    - Artifact repeatability at NRC
  - Kinds of uncertainties:
    - Type A
    - Type B
    - Uncorrelated
    - Correlated

$$R_{i,j,m} = \frac{I_{\mathrm{v}(i,j)}}{V_{i,j,m}} \left(\frac{\mathrm{cd}}{\mathrm{volt}} \right)$$



CCPR Key Comparison CCPR-K3.2014

Comparison Procedures ● Data Analysis

- Sources of Uncertainty ● combination of uncertainties*
  - Kinds of Uncertainties:
    - Type A
    - Type B
    - Uncorrelated (uc)
    - Correlated (c)

$$Q = f(x_i)$$

$$u_{uc}^2(Q) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 \cdot u_{uc}^2(x_i)$$

$$u_c^2(Q) = \left[\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right) \cdot u_c(x_i)\right]^2$$

$$u_{total}^2(Q) = u_{uc}^2(Q) + u_c^2(Q)$$

- *GUM, *Guides to the expression of uncertainty in measurement*, JCGM 100:2008, etc. www.bipm.org

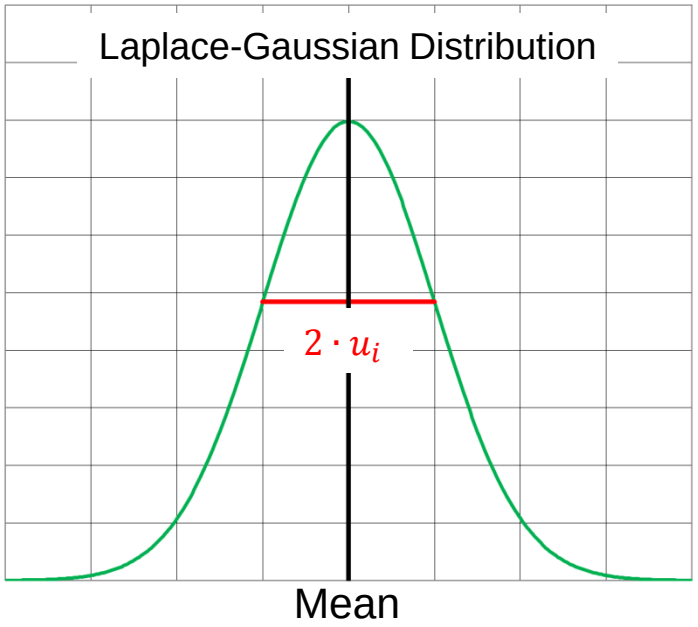
CCPR Key Comparison CCPR-K3.2014

Comparison Procedures ● Data Analysis

- Sources of Uncertainty ● combination of uncertainties ● weighted mean
  - Weights $\mathbf{W}_i = \frac{1}{u_i^2}$
  - Normalised $w_i = \frac{\mathbf{W}_i}{\sum \mathbf{W}_i}$

$$Q = \sum_{i=1}^{n} w_i \cdot x_i$$

$$\frac{\partial Q}{\partial x_i} = w_i$$



CCPR Key Comparison CCPR-K3.2014

Comparison Procedures ● Data Analysis

- Sources of Uncertainty ● combination of uncertainties ● weighted mean
  - Type A
  - Type B
  - Uncorrelated
  - Correlated

Measured Quantity	Uncertainty		
	Type A	Type B	Combined
V_1	$u_A(V_1)$	$u_B(V_1)$	$\sqrt{u_A^2(V_1) + u_B^2(V_1)}$
V_2	$u_A(V_2)$	$u_B(V_2)$	$\sqrt{u_A^2(V_2) + u_B^2(V_2)}$
....	....	....	....
V_n	$u_A(V_n)$	$u_B(V_n)$	$\sqrt{u_A^2(V_n) + u_B^2(V_n)}$
$f(V_i) = \sum_{i=1}^{n} w_i V_i$	$u_A^2(f) = \sum_{1}^{n} w_i^2 u_A^2(V_i)$	$u_B^2(f) = \left[\sum_{1}^{n} w_i u_B\ (V_i)\right]^2$	$\sqrt{u_A^2(f) + u_B^2(f)}$
Weighted mean	Uncorrelated	Correlated	Combined

CCPR Key Comparison CCPR-K3.2014

$$R_{i,j,m} = \frac{I_{\mathrm{v}(i,j)}}{V_{i,j,m}} \left(\frac{\mathrm{cd}}{\mathrm{volt}} \right)$$

Comparison Procedures ● Analysis

- Data analysis
  - Determine final NRC measurement value for each artifact: $R_{i,j} = \langle R_{i,j,m} \rangle_m$, ~12x6=72 values
    - $u(R_{i,j})$ is a combination of NRC measurements (u_A and u_B), Participant (u_A and u_B) and lamp u_A
  - Determine final NRC measurement value for each participant: $R_i = \langle R_{i,j} \rangle_j$, = 12 values
    - $u(R_i)$ is a combination of the (u_A and u_B) components of $u(R_{i,j})$
- Comparison of participant SI candela realisations
  - KCRV (Key Comparison Reference Value)
    - Weighted mean with cut-off
  - Identification of ‘outliers’: deviation from KCRV greater than 6 times their uncertainty
  - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

- Comparison of participant SI candela realisations
  - KCRV (Key Comparison Reference Value)
    - Weighted mean with cut-off

$$u_{cut-off} = average(u_7 \text{ to } u_{12})$$

Participant Luminous Intensity uncertainty Relative standard values (ordered highest to lowest)	
unadjusted	adjusted $u_{adj}(NMI)$
u_1	u_1
u_2	u_2
....	....
u_6	u_6
median	
u_7	u_7
....	....
u_j	u_j
u_k	u_{cutoff}
....	u_{cutoff}
u_{11}	u_{cutoff}
u_{12}	u_{cutoff}

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

- Comparison of participant SI candela realisations
  - KCRV (Key Comparison Reference Value)
    - Weighted mean with cut-off

$$u_{adj}^2(R_i) = u_{adj}^2(NMI) + u_{transfer}^2(R_i)$$

weights $\mathbf{W}_{i,adj} = \frac{1}{u_{adj}^2(R_i)}$

normalised $w_{i,adj} = \frac{\mathbf{W}_{i,adj}}{\sum \mathbf{W}_i,adj}$

Participant Luminous Intensity uncertainty Relative standard values (ordered highest to lowest)	
unadjusted	adjusted $u_{adj}(NMI)$
u_1	u_1
u_2	u_2
....	....
u_6	u_6
median	
u_7	u_7
....	....
u_j	u_j
$u_{cut-off}$	
u_k	u_{cutoff}
....	u_{cutoff}
u_{11}	u_{cutoff}
u_{12}	u_{cutoff}

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

- Comparison of participant SI candela realisations
  - KCRV (Key Comparison Reference Value)
    - Weighted mean with cut-off

$$R_{KCRV} = \sum_{i=1}^{n} w_{i,adj} \cdot R_i \left(\frac{\text{cd}}{\text{volt}} \right)$$

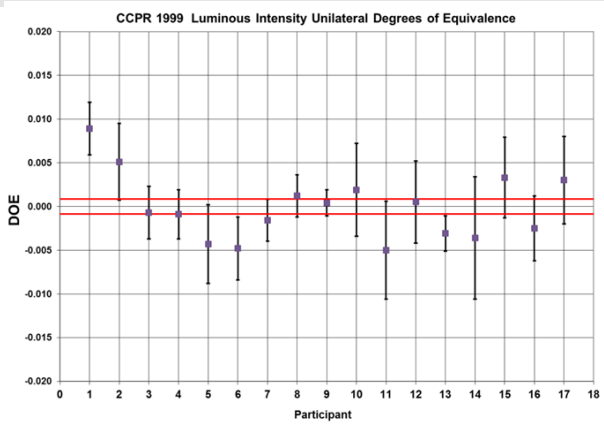
$$u^2(R_{KCRV}) = \sum_{i=1}^{n} w_{i,adj}^2 \cdot u^2(R_i)$$

(uncorrelated)

Participant Luminous Intensity uncertainty Relative standard values (ordered highest to lowest)	
unadjusted	adjusted $u_{adj}(NMI)$
u_1	u_1
u_2	u_2
....	....
u_6	u_6
u_7	u_7
....	....
u_j	u_j
u_k	u_{cutoff}
....	u_{cutoff}
u_{11}	u_{cutoff}
u_{12}	u_{cutoff}

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures ● Analysis



- Comparison of participant SI candela realisations
  - KCRV (Key Comparison Reference Value)
    - Weighted mean with cut-off
  - Identification of ‘outliers’: deviation from KCRV greater than 6 times their (k=1) uncertainty
  - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$

$$\chi^2_{observed} = \sum_{i=1}^{n} \frac{(R_i - R_{KCRV})^2}{u^2_{adj}(R_i)}$$

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures ● Analysis

- Comparison of participant SI candela realisations
  - KCRV (Key Comparison Reference Value)
    - Weighted mean with cut-off
  - Identification of ‘outliers’: deviation from KCRV greater than 6 times their uncertainty
  - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$

IF $\chi^2_{observed} > \chi^2_{0.05}(\nu)$ (inconsistent!)

THEN add Mandel-Paule adjustment uncertainty *s*

$$u^2_{adj}(R_i) = u^2_{adj}(NMI) + u^2_{transfer}(R_i) + s^2$$

And REPEAT calculations with various *s* until ‘consistent’

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures ● Analysis

- Comparison of participant SI candela realisations
  - KCRV (Key Comparison Reference Value)
    - Weighted mean with cut-off
  - Identification of ‘outliers’: deviation from KCRV greater than 6 times their uncertainty
  - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$
  - Calculate the Unilateral Degrees of Equivalence (DOE): D_i

$$D_i = \frac{R_i - R_{KCRV}}{R_{KCRV}}$$

$$u_i^2 = u^2(R_i) + u^2(R_{KCRV}) - 2(w_i \cdot u^2(R_i))$$

R_i *and* R_{KCRV} *are correlated*

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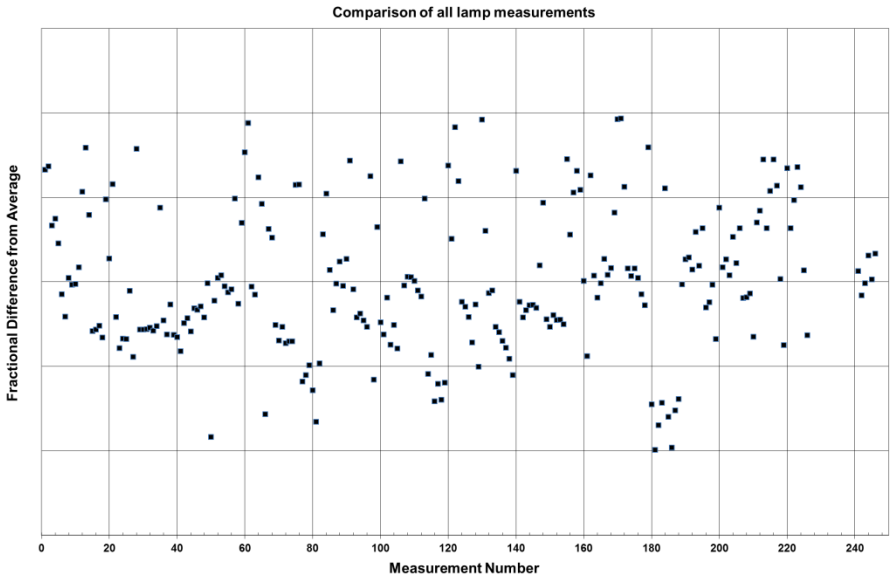
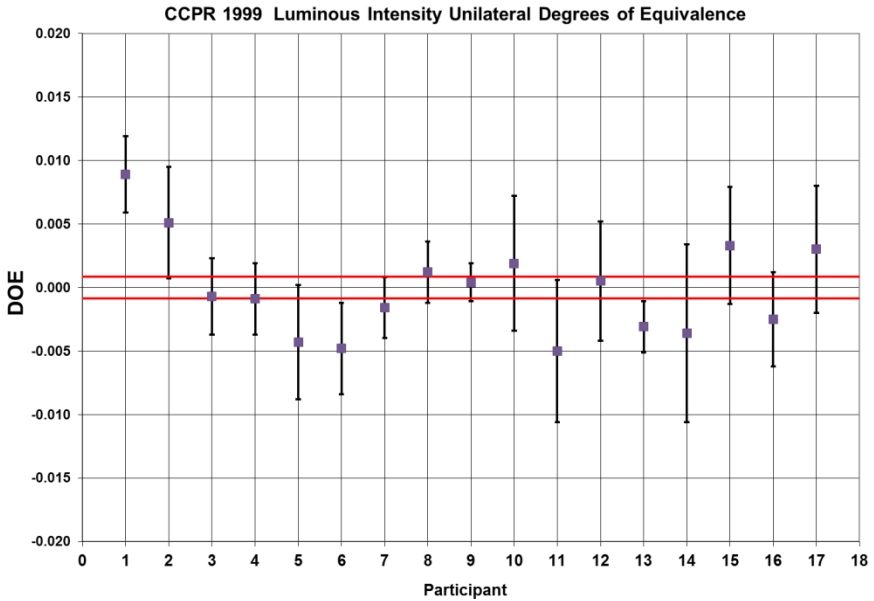
Comparison Procedures ● Data Analysis





CCPR Key Comparison CCPR-K3.2014

Comparison Procedures ● Analysis

- Comparison of participant SI candela realisations
  - KCRV (Key Comparison Reference Value)
    - Weighted mean with cut-off
  - Identification of ‘outliers’: deviation from KCRV greater than 6 times their uncertainty
  - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$
  - Calculate the Unilateral Degrees of Equivalence (DOE)
  - Calculate the Bilateral Degrees of Equivalence

$$D_{i,j} = \frac{R_i - R_j}{R_{KCRV}}$$

$$u^2_{i,j} = u^2(R_i) + u^2(R_j)$$

(R_i and R_j uncorrelated)

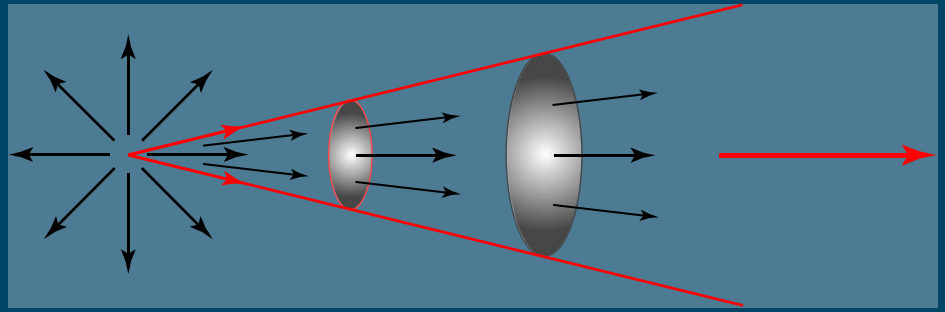
CCPR Key Comparison CCPR-K3.2014

Comparison Organisation

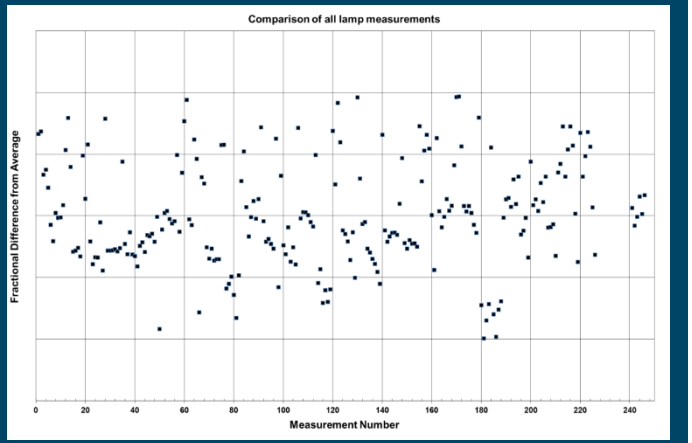
- Selection of participants, artifacts and protocol

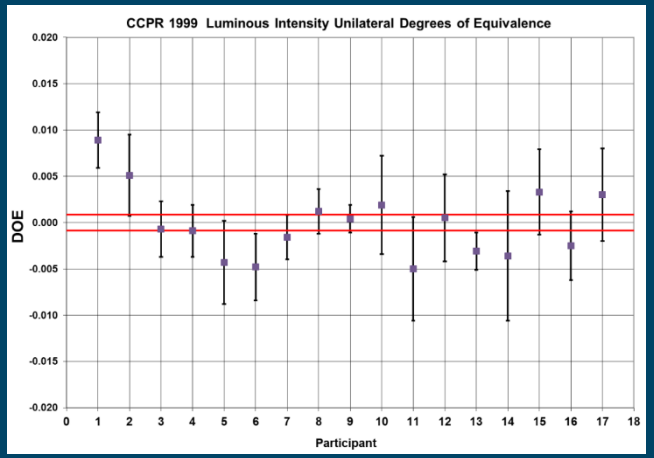
Comparison Procedures

- Comparison measurements and measurement verification
- Data analysis and comparison of participant SI candela realisations
- Write the report
  - Draft A and any revisions, confidential to participants
  - Draft B to CCPR WG-KC for approval (and/or any revisions)
  - Approved Draft B to CCPR for approval
  - Final Report



CCPR KEY COMPARISON CCPR-K3.2014





ACKNOWLEDGEMENTS

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THANK YOU

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